Numerical Inversion of Seismic Surface Wave Dispersion Data and Crust-Mantle Structure in the New York–Pennsylvania Area[1]

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Abstract. A least-squares curve-fitting program in which calculations of surface wave dispersion are used has been developed in order to compute an interpretation of empirical dispersion data in terms of a layered model of the earth. Input may consist of dispersion data (phase velocity versus wave period) for both Love and Rayleigh waves in any modes of propagation. Output consists of successive approximations of the values of parameters such as layer thicknesses, shear velocities, and densities which minimize the mean square of residuals of the empirical data with respect to theoretical dispersion curves calculated from the parameters. Tests made by applying the method to precisely computed theoretical dispersion curves demonstrate the validity of the method. In these, as many as six parameters of the original theoretical models are calculated under a wide variety of conditions from the dispersion data only. In other cases, the amount of information obtainable may be greater or smaller, depending on the quality of the input data.

The New York–Pennsylvania dispersion data of *Oliver et al.* [1961] can be successfully interpreted by using additional information from seismic refraction and studies of nearby earthquakes in the area. A solution including the important effect of the mantle low-velocity channel gives a crustal thickness of 38.6 km, a crustal shear velocity of 3.64 km/sec, a shear velocity of 4.685 km/sec below the M discontinuity, and a ratio of 2.86/3.30 between crustal and subcrustal density values. The density measurement is a new result. The shear velocity structure derived here is consistent with results obtained by Katz from seismic refraction profiles in the same area. Additional data are needed in order to derive more detailed information on crustal structure.

INTRODUCTION

This paper describes the first results obtained by an objective data-processing method for interpreting phase velocity dispersion data for Love and Rayleigh waves in terms of earth structures. In the following section the theory of this computation is defined. A program to implement this theory has been written and tested in a large-scare digital computer. A preliminary report on this work was given earlier [*Dorman and Ewing,* 1962]. Numerical results discussed in the present paper have been obtained for problems concerned with interpretation of the Rayleigh wave dispersion data of *Oliver et al.* [1961, hereinafter referred to as paper I] for the New York–Pennsylvania area.

In recent years observations of surface wave dispersion have provided important data on the structure of the interior of the earth. The usual method of interpretation is an indirect trial-and-error procedure in which, by repetitive use of a method of computing the theoretical dispersion curves (phase velocity versus period) for hypothetical earth models, a theoretical dispersion curve for a model is found that fits the observed data. Several theoretically exact methods have been developed for making this dispersion computation under quite general conditions [see *Haskell,* 1953; *Satô,* 1959; *Alterman et al.,* 1959]. In studying the earth by means of surface wave dispersion data, however, the problem

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to be solved is the inverse of the dispersion problem; i.e., to find from a given set of observed dispersion data the corresponding earth model. Apparently, all formal solutions of the surface wave inversion problem involve simplifying mathematical approximations, and none have been applied extensively to experimental data. Nevertheless, it is important to note some theoretical results. *Knopoff* [1961] found, with regard to the difficult problem of uniqueness, that in the inversion of Love wave data 'any dispersion curve can correspond to an infinity of possible distributions of density and modulus.' It seems reasonable to suppose that this result applies to Rayleigh waves as well. This explains the relative simplicity of finding an earth model corresponding to a single dispersion curve. A remaining question of great practical importance, for which no theoretical answer has been given, is whether the ambiguity noted by Knopoff is removed by using simultaneous dispersion data in more than one mode (Rayleigh and Love modes, fundamental and higher modes, etc.). No inversion theory has been offered previously which permits use of simultaneous data in more than one mode. Moreover, the practical difficulty of finding such a solution by trial-and-error inversion is very great.

The purpose of this paper is to offer a method of finding the desired inverse solution on the basis of a known method of dispersion calculation combined with the well-known technique of curve fitting by least-squares analysis. Initially we have a set of experimentally observed phase velocity dispersion data, and we assume a preliminary, approximate, layered earth model. The phase velocity for the model is then calculated theoretically by the method of Haskell matrices at the period of each observed phase velocity datum. Also, by dispersion calculations we obtain theoretical values of derivatives of the form $\partial c/\partial p$, where c is a theoretical phase velocity and p is a parameter of a layered structure. A rectangular array of these derivatives is calculated which covers the phase velocities at the period of each observed phase velocity datum and covers each parameter that is to be evaluated. The array of derivatives is the matrix of coefficients of linear equations relating the desired parameter corrections to the differences between the corresponding theoretical and observed phase velocity values. Terms of order higher than the first are neglected. If the number of equations or the number of observed data exceeds the number of parameters, as in the cases treated here, a least-squares evaluation of the parameter corrections is possible. After several repetitions of this process the corrections approach zero. The running time required to perform this computation can be estimated by an expression given below. Problems described in this paper require between ½ minute and 3 minutes per repetition on the IBM 7090.

This inversion method replaces the trial-and-error selection of corrections. It produces results which are exact in the sense that the earth parameters used in an exact theoretical calculation of dispersion data can be recovered from a knowledge of the dispersion data alone. The method permits a survey of the possible interpretations of a given set of data which is more rapid, precise, and complete than can be made by trial and error. Also, by observing the performance of the calculation under various sets of constraining conditions, a practical evaluation of the uniqueness question in each case can be made. The number of variables that can be evaluated simultaneously depends on the quality of the data. The number of modes observed, the breadth of the observed spectrum in each mode, and the small scatter of the data points are quality factors which strongly affect the performance of the calculation. In the examples given in this paper the results obtainable appear to be limited by the quality of the data rather than by the statistical method or numerical implementation of the method.

The problems solved herein by means of the present inversion method are related to a study of the structure of the New York–Pennsylvania area using the Rayleigh wave phase velocity data of paper I. These dispersion data, covering periods between 16 and 45 seconds, were derived by the tripartite method from seismograms of four earthquakes recorded on matched long-period instruments at Palisades, New York; Waynesburg, Pennsylvania; and Ottawa, Canada. Fifty-eight data points used in this paper are those shown in Figure 7 of paper I. These data are also listed in Table 4 of this paper. The reader is referred to paper I for details of the derivation of these data. In paper I these data were interpreted by trial-and-error inversion, the best solution being case 8123 for three crustal layers.

To test the performance of the inversion program when the answer is known exactly, several numerical problems described below were solved using dispersion data computed theoretically for a hypothetical layered structure and covering the same range of periods as the New York–Pennsylvania data. Results of these problems therefore indicate the optimum performance to be expected and the maximum amount of information obtainable from the New York–Pennsylvania data. In ideal problems with data like these it appears possible to recover at least four and perhaps six parameters in a single-layered crustal configuration. Uniqueness is strongly suggested in these theoretical problems by sucessful recovery of the correct values from several sets of initial conditions. In this context uniqueness is taken to mean the existence of only one solution (a set of parameters satisfying the least-squares condition) within the framework of layering provided in the initial case and within a considerable range of values of the parameters.

The results of inversion calculations on the data of paper I show that models involving only one crustal layer can yield inversion solutions that are superior to case 8123 in the sense that their dispersion curves agree better with the experimental data. Also, some of these models agree well with Katz's one-layer interpretations of crustal structure from seismic refraction profiles in the same region [*Katz*, 1955]. On the other hand, in trying to find inversion solutions similar to case 8123, the rather large number of parameters required to describe three crustal layers apparently are not subject to a unique interpretation on the basis of present surface wave data only.

Data of *Katz* [1955] on crustal and subcrustal compressional velocities are used as fixed values throughout the calculations. Data of Lehmann on the shear velocity structure of the upper mantle in this region [see *Lehmann*, 1955, 1961; *Dorman et al.*, 1960] have been used in order to obtain more accurate results for crustal structure. The New York–Pennsylvania data do not contain long enough wavelengths to permit an independent determination of upper mantle structure; nevertheless, the longer wavelengths are affected significantly by the low-velocity channel of the Lehmann model.

In a new paper *Brune and Dorman* [1963] have applied the numerical inversion method to the interpretation of data on Love and Rayleigh wave dispersion for the Canadian shield. These data have a broad spectrum, periods of 3 to 90 seconds, and in addition carry information in a mode, the fundamental Love mode, that is not represented in the New York–Pennslyvania data. In the Canadian case, superior definition of the experimental curves is evidenced by the fact that the phase velocity residuals with respect to the dispersion curves for the final model, case CANSD, have an rms value of 0.015 km/sec as compared with 0.040 km/sec in the New York–Pennsylvania case. These favorable circumstances permit a more detailed and precise solution to be determined by the inversion calculation on the Canadian shield data and allow the program to perform in a way that is not as seriously limited by the quality of the data as in the New York–Pennsylvania case. In several computer runs covering different frequency bands it was possible to evaluate eight velocity parameters representing the structure of the crust and the mantle of the Canadian shield. The results show that information of considerable geologic interest can be obtained from dispersion data by the method of numerical inversion in cases where good data are available.

List of Symbols

h_m, thickness in km of layer m.

α_m, compressional velocity in km/sec of layer m.

β_m, shear velocity in km/sec of layer m.

ρ_m, density in g/cm^3 of layer m.

T, wave period in seconds.

c, phase velocity in km/sec.

s, sample standard deviation of experimental phase velocities with respect to theoretical phase velocities, in km/sec.

n, a subscript indicating the index number of the bottom layer or half-space.

e, a subscript applied to c or T above, indicating an experimentally observed value.

i, a subscript indexing the experimentally observed points.

I, the total number of experimental points used in a particular problem.

v, an active parameter which may be a particular h, α, β, or ρ.

j, a subscript indexing the active parameters.

J, the total number of active parameters used in a particular problem.

V, the vector of J active parameters.

dv, a correction to be added to an active parameter.

D, the vector of J corrections.

P, a matrix of partial derivatives. It has I rows and J columns.

C, a vector of residues. It has I elements.

METHOD OF THE INVERSION CALCULATION

Using the multilayer dispersion computation [*Haskell*, 1953; also see *Dorman*, 1962] we have written a subroutine which numerically evaluates the function

$$c = f(T, h_1, \alpha_1, \beta_1, \rho_1, h_2, \alpha_2, \beta_2, \rho_2, \cdots, \alpha_n, \beta_n, \rho_n) \quad (1)$$

upon substitution of numerical values for the arguments enclosed in parentheses (see list of symbols). In this computation the earth is an elastic half-space of flat homogeneous layers, the subscript n denoting the bottom, semi-infinite medium. Experimental dispersion data are a set of observed wave periods together with corresponding observed phase velocities (T_{e1}, c_{e1}), (T_{e2}, c_{e2}), . . . , (T_{eI}, c_{eI}). If T in (1) is regarded as the independent variable and the remaining arguments are considered to be parameters of the curve c versus T, the inversion problem can be stated as the problem of choosing the parameters so that differences between observed and theoretical phase velocities at observed periods satisfy the least-squares condition

$$\sum_{i=1}^{I} (c_{ei} - c_i)^2 = \text{minimum} \quad (2)$$

where $c_i = f(T_{ei}, \ldots)$ from (1). If **C** is defined as a column vector which has I elements, $c_{ei} - c_i$, then in matrix notation (2) can be written

$$\tilde{\mathbf{C}}\mathbf{C} = \text{minimum} \quad (2a)$$

where the tilde denotes the transpose of a vector or a matrix. Thus the inversion problem is one of curve fitting, where the approximating function is (1) as defined by the multilayer calculation rather than a power series or some other commonly used function of statistics. When condition 2 is satisfied the parameters of the approximating function have direct physical significance as the description of the layered earth which best corresponds to the given dispersion data. In a particular inversion problem a suitable form of the approximating function is chosen by selecting an appropriate number of layers, n, and by other options allowed in the digital computer program as described below. In the present form of the program n remains fixed throughout the execution of a problem. As is indicated in (1), the total number of parameters necessary for the multilayer computation is $4n - 1$, except for computations on Love wave data exclusively, which require only $3n - 1$ parameters, the compressional velocities being immaterial.

Since $4n - 1$ is usually too large a number of parameters to form a useful approximating function, the inversion program has facilities for dividing the $4n - 1$ parameters into an 'active' and a 'passive' group at the option of the user. Assignment of a parameter to one of these groups is made at the beginning of a problem and holds throughout. With an initial set of parameter values chosen according to available geophysical data, the active parameters are subject to revision by the inversion program according to (2), while the passive parameters remain fixed at their initial values.

Geophysical considerations serve to limit the number of parameters which are placed in the active group by eliminating those for which sparse information is available. In this paper, for instance, we will not attempt to evaluate layer compressional velocities, since surface waves are not very sensitive to them. Also, densities are less important than shear velocities and in some cases may be taken from an independent density versus depth law with little effect on shear velocity results. In many cases it will be useful to assume that the earth consists of n layers of fixed thicknesses, compressional velocities, and densities and to attempt to evaluate layer shear velocities only. In this case there would be n active parameters. Period equations for Rayleigh and Love waves can be expanded in a form in which a particular layer density appears only as a ratio with the density of an adjacent layer. Therefore, layer densities can be determined only to a common factor, or, alternatively, $n-1$ layer densities at most can be determined independently. For data covering a rather limited spectrum, as in this paper, a common problem will be to determine the best crustal solution consisting of a single surface layer overlying a half-space. This involves the evaluation of h_1, β_1, β_2, and ρ_1/ρ_2. Thus in most problems the number of active parameters will be considerably less than $4n-1$.

The initial values of the active parameters, $v_1, v_2, \ldots, v_J$, may be represented by the column vector, $\mathbf{V}$. A set of corrections $dv_1, dv_2, \ldots, dv_J$, denoted by the column vector, $\mathbf{D}$, is sought so that, when $\mathbf{V}$ is replaced by $\mathbf{V} + \mathbf{D}$, (2) is satisfied. We will rewrite (1) as

$$c = f(T, v_1, v_2, \ldots, v_J) \qquad (1a)$$

where the passive parameters are not mentioned explicitly as arguments. Thus $c_i = f(T_{ei}, v_1, v_2, \ldots, v_J)$. A small increment, Δc_{ij}, at period T_{ei} produced by a small increment Δv_j in v_j can then be evaluated numerically using ($1a$) as follows:

$$\Delta c_{ij} = f(T_{ei}, v_1, v_2, \ldots, \cdot v_j + \Delta v_j, \ldots, v_J) - c_i$$

We then calculate the partial derivative of c at period T_{ei} with respect to v_j as

$$\partial c_i/\partial v_j = \Delta c_{ij}/\Delta v_j \qquad (3)$$

Using partial derivatives evaluated at the period T_{ei} we may write the following linear equation of condition, relating the required corrections dv_j to the observed residual $c_{ei} - c_i$:

$$\frac{\partial c_i}{\partial v_1} dv_1 + \frac{\partial c_i}{dv_2} dv_2 + \cdots \frac{\partial c_i}{\partial v_J} dv_J = c_{ei} - c_i \qquad (4)$$

A similar equation of condition corresponding to each observed period can be formed. Then, using matrix notation, we write the I equations of condition as

$$\mathbf{PD} = \mathbf{C} \qquad (4a)$$

where $\mathbf{P}$ is the matrix of partial derivatives with I rows and J columns. $\mathbf{D}$ represents a column vector of J elements which are the dv_j's. From ($4a$) we can obtain the corresponding normal equations

$$\tilde{\mathbf{P}}\mathbf{PD} = \tilde{\mathbf{P}}\mathbf{C} \qquad (5)$$

These equations are solved for $\mathbf{D}$. $\mathbf{D}$ is then the well-known least-squares solution of ($4a$) [see *Margenau and Murphy*, 1943, p. 500]. Thus

TABLE 1 - CASE TH1

	DATA	INITIAL	1ST CORR	2ND CORR	3RD CORR
PARAMETERS					
h_1	38.00000	38.00000	38.05446	37.99766	38.00000*
α_1	6.14000	6.14000	6.14000	6.14000	6.14000
β_1	3.55000	3.45000	3.54982	3.54998	3.55000*
ρ_1	2.74000	3.00000	2.76118	2.74014	2.74000*
α_2	8.20000	8.20000	8.20000	8.20000	8.20000
β_2	4.65000	4.75000	4.65113	4.64998	4.65000*
ρ_2	3.32000	3.32000	3.32000	3.32000	3.32000
T	C	C_e-C	C_e-C	C_e-C	C_e-C
RAYLEIGH MODE					
45.0	4.02677	-0.00541	0.00302	0.00002	0.00000
36.0	3.93010	0.02165	0.00412	0.00001	0.00000
32.0	3.85351	0.04089	0.00456	-0.00000	0.00000
30.0	3.80322	0.05203	0.00465	-0.00001	0.00000
28.0	3.74408	0.06329	0.00458	-0.00002	0.00000
26.0	3.67688	0.07344	0.00428	-0.00002	0.00000
24.0	3.60429	0.08121	0.00376	-0.00002	0.00000
22.0	3.53070	0.08587	0.00308	-0.00002	0.00000
20.0	3.46120	0.08760	0.00235	-0.00002	0.00000
18.0	3.40013	0.08723	0.00167	-0.00001	0.00000
16.0	3.35025	0.08576	0.00112	-0.00001	0.00000
RMS=	0.00000	0.06801	0.00358	0.00001	0.00000

after the operation

$$\text{replace } \mathbf{V} \text{ by } \mathbf{V} + \mathbf{D} \qquad (6)$$

the function (1a), formed with the new parameters $\mathbf{V}$, satisfies (2) approximately. The derivatives (3) are themselves functions of c_i and all the parameters. Therefore, linear equations of the type of (4a) are accurate only when the dv_j's are small. Repetition of the process represented by (3), (4), (4a), (5), and (6), using the new values of the parameters as initial values, leads to a further approximation of the desired solution $\mathbf{V}$. The function $\tilde{\mathbf{C}}\mathbf{C}$, which is calculated after each correction, is minimized after a few repetitions.

$\mathbf{V}$ is then a unique solution of the experimental dispersion data in the sense that there is no other vector in the local region of $\mathbf{V}$ which satisfies (2a) as well, subject to the given constraints implicit in: (1) the number of layers chosen for the problem; (2) the particular division made between active and passive parameters; and (3) the fixed values adopted for the passive parameters. If the data and the constraints are such that no unique solution $\mathbf{V}$ exists, the successive $\mathbf{V}$'s may wander through a series of values which are equally good in the sense that $\tilde{\mathbf{C}}\mathbf{C}$ does not change significantly from one approximation to another. If the first moment of the residuals is very nearly zero, after minimizing $\tilde{\mathbf{C}}\mathbf{C}$, the final value of $s = \sqrt{\tilde{\mathbf{C}}\mathbf{C}}$ is properly termed the standard deviation of the sample and is a measure of the scatter of the experimental phase velocity data. Precision measures for the elements of $\mathbf{V}$ are not calculated formally, but useful estimates of these can be made by physical considerations based on data produced by the inversion calculation, as shown in the cases below.

Numerical Calculations

A digital computer program to implement the above theory has been written and tested on the IBM 7090. Technical details of the program will not be discussed herein because the program is still undergoing revision. It is known at present that the portions involving the calculation of roots of the multilayer period equation and of

TABLE 2 - CASE TH2

PARAMETERS	DATA	INITIAL	1ST CORR	2ND CORR	3RD CORR	4TH CORR	5TH CORR
h_1	36.00000	36.00000	35.96187	35.94891	36.00917	36.01146	35.99918*
α_1	6.15000	6.15000	6.15000	6.15000	6.15000	6.15000	6.15000
β_1	3.55000	3.45000	3.54916	3.54928	3.55015	3.55018	3.55004*
ρ_1	2.74000	3.00000	2.78267	2.74958	2.73833	2.73795	2.74057*
h_2	84.00000	84.00000	84.00000	84.00000	84.00000	84.00000	84.00000
α_2	8.14000	8.14000	8.14000	8.14000	8.14000	8.14000	8.14000
β_2	4.70000	4.80000	4.70781	4.70213	4.69963	4.69954	4.70016*
ρ_2	3.30000	3.30000	3.30000	3.30000	3.30000	3.30000	3.30000
h_3	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000
α_3	8.17000	8.17000	8.17000	8.17000	8.17000	8.17000	8.17000
β_3	4.30000	4.30000	4.30000	4.30000	4.30000	4.30000	4.30000
ρ_3	3.44000	3.44000	3.44000	3.44000	3.44000	3.44000	3.44000
α_4	8.49000	8.49000	8.49000	8.49000	8.49000	8.49000	8.49000
β_4	4.76000	4.76000	4.76000	4.76000	4.76000	4.76000	4.76000
ρ_4	3.53000	3.53000	3.53000	3.53000	3.53000	3.53000	3.53000

T	C	$C_e - C$	$C_e - C$	$C_e - C$	$C_e - C$	$C_e - C$	$C_e - C$
RAYLEIGH	MODE						
45.0	4.02246	0.00095	0.00171	-0.00001	0.00000	0.00000	-0.00001
36.0	3.96486	0.01684	0.00293	0.00003	-0.00001	-0.00001	0.00001
32.0	3.90522	0.03301	0.00367	0.00003	-0.00001	0.00000	0.00000
30.0	3.86170	0.04380	0.00397	0.00002	0.00000	0.00000	0.00001
28.0	3.80703	0.05580	0.00410	-0.00002	-0.00001	0.00000	0.00000
26.0	3.74097	0.06771	0.00398	-0.00006	-0.00001	0.00000	-0.00001
24.0	3.66547	0.07787	0.00359	-0.00009	0.00000	0.00000	0.00000
22.0	3.58497	0.08474	0.00294	-0.00009	-0.00002	0.00001	-0.00001
20.0	3.50579	0.08811	0.00231	-0.00002	0.00000	0.00004	-0.00002
18.0	3.43399	0.08845	0.00166	0.00010	0.00000	0.00000	-0.00002
16.0	3.37395	0.08706	0.00124	0.00022	0.00001	-0.00005	-0.00001
RMS=	0.00000	0.06565	0.00308	0.00009	0.00001	0.00002	0.00001

the variable, s, are entirely correct and in agreement with previous programs. This is sufficient to ensure the validity of any answer obtained which has a low value of s. The running time on the IBM 7090 required for each revision or iteration is approximately 0.015 $(J + 1)$ tnI seconds, where n, I, and J have the same significance as was given above and t is the number of trial solutions required to solve the phase velocity dispersion equation for c within $c \cdot 10^{-6}$ km/sec at a particular period T. t is usually 7 or 8. For example, problem Xp3, described below, required about 2 minutes per iteration and problem Th1 required about 25 seconds per iteration. The execution of some successful problems is discussed below.

Theoretical problem. Theoretical dispersion calculations were made for several crust-mantle models chosen as approximations of the structure of the New York–Pennsylvania area. Rayleigh wave phase velocity data for these models were computed in the same range of periods covered by the experimental New York–Pennsylvania data. These theoretical dispersion data were then used as experimental data in inversion problems. In each problem four to six of the parameters of the structure were chosen as active parameters. The initial values of these parameters were displaced from the values used in computing the theoretical data while the remaining, passive parameters retained their original values. Thus, the constraints of each problem were such that all the parameters of the structure could assume the values that were originally used in the data calculation, provided that the inversion calculation produced the proper corrections for the active parameters. A satisfactory result was obtained in each of the three problems, Th1, Th2, and Th3, shown in Tables 1, 2, and 3.

Problem Th1, involving a single-layered crust overlying a half-space, is shown in detail in

TABLE 3 - CASE TH3

PARAMETERS	DATA	INITIAL	1ST CORR	2ND CORR	3RD CORR	4TH CORR	5TH CORR
h_1	18.00000	18.00000	22.04542	23.50048	14.60667	16.52196	17.73880*
α_1	6.15000	6.15000	6.15000	6.15000	6.15000	6.15000	6.15000
β_1	3.55000	3.45000	3.59965	3.37217	3.53530	3.57852	3.55218*
ρ_1	2.74000	3.00000	1.95210	2.17243	2.69334	2.79085	2.72864*
h_2	18.00000	18.00000	18.00000	18.00000	18.00000	18.00000	18.00000
α_2	6.15000	6.15000	6.15000	6.15000	6.15000	6.15000	6.15000
β_2	3.55000	3.80000	2.75801	3.23600	3.02847	3.30571	3.45145*
ρ_2	2.74000	3.00000	4.36655	3.46258	3.91788	3.18283	2.93249*
h_3	84.00000	84.00000	84.00000	84.00000	84.00000	84.00000	84.00000
α_3	8.14000	8.14000	8.14000	8.14000	8.14000	8.14000	8.14000
β_3	4.70000	4.80000	4.83581	4.75796	4.90299	4.74450	4.72383*
ρ_3	3.30000	3.30000	3.30000	3.30000	3.30000	3.30000	3.30000
h_4	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000
α_4	8.17000	8.17000	8.17000	8.17000	8.17000	8.17000	8.17000
β_4	4.30000	4.30000	4.30000	4.30000	4.30000	4.30000	4.30000
ρ_4	3.44000	3.44000	3.44000	3.44000	3.44000	3.44000	3.44000
α_5	8.49000	8.49000	8.49000	8.49000	8.49000	8.49000	8.49000
β_5	4.76000	4.76000	4.76000	4.76000	4.76000	4.76000	4.76000
ρ_5	3.53000	3.53000	3.53000	3.53000	3.53000	3.53000	3.53000

T	C	$C_e - C$	$C_e - C$	$C_e - C$	$C_e - C$	$C_e - C$	$C_e - C$
RAYLEIGH MODE							
45.0	4.02246	-0.01794	0.05474	0.01977	-0.00629	0.01078	0.00126
36.0	3.96486	-0.02787	0.07476	0.02898	-0.00874	0.01397	0.00169
32.0	3.90522	-0.03777	0.09460	0.03983	-0.00552	0.01713	0.00241
30.0	3.86170	-0.04526	0.10766	0.04782	-0.00138	0.01939	0.00297
28.0	3.80703	-0.05466	0.12174	0.05750	0.00563	0.02223	0.00369
26.0	3.74097	-0.06537	0.13447	0.06816	0.01657	0.02568	0.00460
24.0	3.66547	-0.07550	0.14290	0.07861	0.03212	0.02962	0.00573
22.0	3.58497	-0.08217	0.14476	0.08759	0.05135	0.03345	0.00692
20.0	3.50579	-0.08240	0.14015	0.09475	0.07160	0.03645	0.00801
18.0	3.43399	-0.07460	0.13106	0.10088	0.08936	0.03775	0.00877
16.0	3.37395	-0.05902	0.12026	0.10701	0.10218	0.03670	0.00898
RMS=	0.00000	0.06040	0.11858	0.07231	0.05016	0.02736	0.00569

TABLE 4 - CASE XP1

DATA PARAMETERS		INITIAL	1ST CORR	2ND CORR	3RD CORR	4TH CORR	5TH CORR
h_1		38.00000	43.67011	43.10487	43.44307	43.24995	43.36303*
α_1		6.14000	6.14000	6.14000	6.14000	6.14000	6.14000
β_1		3.45500	3.67724	3.67629	3.67975	3.67815	3.67909*
ρ_1		3.00000	1.57746	1.87422	1.86446	1.89229	1.87660*
α_2		8.20000	8.20000	8.20000	8.20000	8.20000	8.20000
β_2		4.50000	4.34020	4.40540	4.41227	4.41762	4.41470*
ρ_2		3.32000	3.32000	3.32000	3.32000	3.32000	3.32000
T	C	C_e-C	C_e-C	C_e-C	C_e-C	C_e-C	C_e-C
RAYLEIGH MODE							
16.3	3.40	0.13441	-0.03109	-0.02941	-0.03028	-0.02985	-0.03009
16.3	3.43	0.16441	-0.00109	0.00059	-0.00028	0.00015	-0.00009
17.2	3.54	0.25615	0.09100	0.09332	0.09280	0.09310	0.09293
17.9	3.46	0.16055	-0.00463	-0.00166	-0.00192	-0.00172	-0.00183
18.1	3.48	0.17587	0.01064	0.01381	0.01363	0.01380	0.01371
18.1	3.50	0.19587	0.03064	0.03381	0.03363	0.03380	0.03371
18.7	3.50	0.18130	0.01571	0.01957	0.01962	0.01972	0.01967
18.8	3.44	1.11879	-0.04689	-0.04290	-0.04281	-0.04272	-0.04276
19.1	3.51	0.18113	0.01517	0.01955	0.01976	0.01980	0.01979
19.5	3.51	0.17063	0.00418	0.00913	0.00948	0.00948	0.00950
19.5	3.54	0.20063	0.03417	0.03913	0.03948	0.03948	0.03950
19.8	3.44	0.09254	-0.07436	-0.06895	-0.06849	-0.06852	-0.06848
20.0	3.50	0.14706	-0.02018	-0.01445	-0.01392	-0.01397	-0.01392
20.2	3.48	0.12149	-0.04610	-0.04004	-0.03944	-0.03951	-0.03945
20.4	3.54	0.17586	0.00788	0.01428	0.01495	0.01486	0.01493
20.7	3.47	0.09728	-0.07134	-0.06440	-0.06363	-0.06375	-0.06366
21.2	3.51	0.12267	-0.04714	-0.03927	-0.03835	-0.03850	-0.03839
21.4	3.60	0.20672	0.03639	0.04466	0.04563	0.04547	0.04559
21.5	3.48	0.08373	-0.08687	-0.07840	-0.07740	-0.07757	-0.07745
21.5	3.53	0.13373	-0.03687	-0.02840	-0.02740	-0.02757	-0.02745
21.8	3.55	0.14468	-0.02676	-0.01769	-0.01661	-0.01679	-0.01666
21.9	3.54	0.13164	-0.04099	-0.03081	-0.02971	-0.02990	-0.02976
22.0	3.60	0.18858	0.01657	0.02605	0.02717	0.02698	0.02712
22.5	3.53	0.10317	-0.07037	-0.05984	-0.05861	-0.05882	-0.05867
22.8	3.62	0.18382	0.00933	0.02050	0.02177	0.02157	0.02172
23.4	3.90	0.14494	-0.03148	-0.01904	-0.01770	-0.01790	-0.01775
23.5	3.66	0.20178	0.02504	0.03769	0.03903	0.03883	0.03898
23.5	3.64	0.18178	0.00504	0.01769	0.01903	0.01883	0.01898
23.7	3.68	0.21544	0.03806	0.05112	0.05248	0.05228	0.05243
24.1	3.67	0.19274	0.01410	0.02797	0.02934	0.02915	0.02929
24.5	3.61	0.12002	-0.05982	-0.04518	-0.04382	-0.04399	-0.04386
24.7	3.69	0.19367	0.01326	0.02827	0.02961	0.02945	0.02958
24.9	3.77	0.26732	0.08637	0.10173	0.10305	0.10291	0.10302
25.5	3.65	0.12837	-0.05402	-0.03772	-0.03649	-0.03659	-0.03650
25.6	3.74	0.21520	0.03263	0.04906	0.05028	0.05019	0.05028
25.8	3.75	0.21897	0.03597	0.05267	0.05385	0.05378	0.05385
26.1	3.71	0.16962	-0.01388	0.00318	0.00428	0.00424	0.00430
26.5	3.69	0.13727	-0.04674	-0.02929	-0.02830	-0.02831	-0.02827
27.0	3.72	0.15205	-0.03231	-0.01452	-0.01368	-0.01365	-0.01364
27.1	3.77	0.19903	0.01464	0.03248	0.03329	0.03333	0.03334
27.2	3.74	0.16603	-0.01837	-0.00049	0.00028	0.00033	0.00033
27.8	3.70	0.10827	-0.07593	-0.05793	-0.05739	-0.05729	-0.05732
28.0	3.78	0.18246	-0.00156	0.01642	0.01687	0.01700	0.01696
28.8	3.74	0.11976	-0.06295	-0.04530	-0.04519	-0.04500	-0.04508
29.5	3.81	0.17069	-0.01013	0.00692	0.00670	0.00693	0.00682
29.7	3.80	0.15539	-0.02478	-0.00795	-0.00826	-0.00802	-0.00814
30.4	3.81	0.14735	-0.03014	-0.01423	-0.01488	-0.01461	-0.01474
31.1	3.83	0.15013	-0.02414	-0.00935	-0.01033	-0.01004	-0.01019
31.8	3.84	0.14375	-0.02685	-0.01333	-0.01463	-0.01434	-0.01450
32.1	3.88	0.17699	0.00808	0.02101	0.01958	0.01987	0.01971
32.6	3.83	0.11605	-0.04991	-0.03798	-0.03964	-0.03935	-0.03951
35.1	3.87	0.10744	-0.04223	-0.03562	-0.03824	-0.03805	-0.03817
35.4	3.95	0.18224	0.03463	0.04059	0.03787	0.03805	0.03793
35.5	3.92	0.15054	0.00361	0.00936	0.00660	0.00678	0.00667
35.9	3.95	0.17387	0.02968	0.03459	0.03170	0.03185	0.03175
44.1	4.04	0.16615	0.07287	0.06375	0.05924	0.05877	0.05899
45.3	3.91	0.02644	-0.06066	-0.07129	-0.07592	-0.07649	-0.07622
45.9	4.01	0.12189	0.03777	0.02642	0.02173	0.02112	0.02142
RMS=		0.16372	0.04168	0.03985	0.03982	0.03981	0.03981

Table 1. The first two columns of the table list the experimental phase velocity data used in the problem and the layer parameters from which they were calculated. Column 3 lists the layer parameters of the initial assumed approximate solution and the residuals of phase velocities for this case with respect to the phase velocity data. Succeeding columns list the layer parameters and residuals for succeeding corrections to the initial solution which were obtained by the inversion program. The last line of the table gives rms residual, s, for each approximation. The program seeks a minimum of this quantity, and its decrease from column to column is a measure of the convergence of the solution. Tables 2 and 3, illustrating cases Th2 and Th3, are constructed similarly. In each table active parameters are identified by asterisks at the ends of the proper rows.

The structure used in problem Th2 has three layers overlying a half-space to represent the four regions: crust, upper mantle, low velocity channel, and deeper mantle. The active parameters chosen for this problem are h_1, β_1, ρ_1, and β_2. The layer configuration for problem Th3 is the same as for Th2 except that the top layer is split into two layers of identical properties. In this problem, six active parameters are chosen instead of four as in problems Th1 and Th2.

Experimental problems. The results of three experimental determinations of crustal structure, using structural configurations of increasing complexity, are shown in Tables 4, 5, and 6 as cases Xp1, Xp2, and Xp3, respectively.

A similar configuration and the same active parameters are used in case Xp1 as in Th1. Initial choices of all parameters were based approximately on the data of Katz and on other geophysical data. Table 4 is similar in format to Table 1, with the exception that the lower parts of columns 1 and 2 of Table 4 contain, respectively, the periods and phase velocities of empirical data (from Figure 7, paper I) rather than theoretical data. The method of computing the data in column 3 and succeeding columns is identical to that used in the theoretical problems. In this case five revisions are enough to approach a limiting value of s of 0.040 km/sec, which represents the scatter of the empirical data.

Tables 5 and 6 contain an upper part and a last line in the format of Table 4, but the information of the lower part of Table 4 is omitted. However, the experimental points used in these cases are exactly the same as those shown in columns 1 and 2 of Table 4.

The initial configuration of layers of case Xp2 is similar to that of case Th2; i.e., it has a single-layered crust overlying a mantle with a low-velocity channel. ρ_1 is omitted from the active parameters as used in Xp1, but the deeper fixed layering approximates the effect of the low-velocity region more accurately than the simple half-space in Xp1. In five revisions s approaches the value 0.040 km/sec, very nearly the same value as in Xp1. But final values of the active parameters differ considerably from those reached in case Xp1, apparently owing to the effect of deeper layering.

Case Xp3 starts from an initial configura-

TABLE 5 - CASE XP2

PARAMETERS	INITIAL	1ST CORR	2ND CORR	3RD CORR	4TH CORR	5TH CORR
h_1	40.00000	40.07664	39.62779	39.54681	39.52263	39.69082*
α_1	6.15000	6.15000	6.15000	6.15000	6.15000	6.15000
β_1	3.50500	3.65076	3.65075	3.64913	3.64871	3.65159*
ρ_1	2.70000	2.70000	2.70000	2.70000	2.70000	2.70000
h_2	84.00000	84.00000	84.00000	84.00000	84.00000	84.00000
α_2	8.14000	8.14000	8.14000	8.14000	8.14000	8.14000
β_2	4.70000	4.65049	4.64406	4.64218	4.64164	4.64619*
ρ_2	3.30000	3.30000	3.30000	3.30000	3.30000	3.30000
h_3	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000
α_3	8.17000	8.17000	8.17000	8.17000	8.17000	8.17000
β_3	4.30000	4.30000	4.30000	4.30000	4.30000	4.30000
ρ_3	3.44000	3.44000	3.44000	3.44000	3.44000	3.44000
α_4	8.49000	8.49000	8.49000	8.49000	8.49000	8.49000
β_4	4.76000	4.76000	4.76000	4.76000	4.76000	4.76000
ρ_4	3.53000	3.53000	3.53000	3.53000	3.53000	3.53000
RMS=	0.10067	0.04037	0.03995	0.03996	0.03996	0.03995

TABLE 6 - CASE XP3

PARAMETERS	INITIAL	1ST CORR	2ND CORR	3RD CORR	4TH CORR	5TH CORR
h_1	36.00000	39.77244	46.41592	48.56155	49.06478	46.50879*
α_1	6.15000	6.15000	6.15000	6.15000	6.15000	6.15000
β_1	3.55500	3.65018	3.70475	3.71226	3.71494	3.70253*
ρ_1	3.00000	2.64250	1.13574	1.04441	0.98396	1.34380*
h_2	84.00000	84.00000	84.00000	84.00000	84.00000	84.00000
α_2	8.14000	8.14000	8.14000	8.14000	8.14000	8.14000
β_2	4.50000	4.62700	4.23352	4.23255	4.22419	4.28730*
ρ_2	3.30000	3.30000	3.30000	3.30000	3.30000	3.30000
h_3	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000
α_3	8.17000	8.17000	8.17000	8.17000	8.17000	8.17000
β_3	4.30000	4.30000	4.30000	4.30000	4.30000	4.30000
ρ_3	3.44000	3.44000	3.44000	3.44000	3.44000	3.44000
α_4	8.49000	8.49000	8.49000	8.49000	8.49000	8.49000
β_4	4.76000	4.76000	4.76000	4.76000	4.76000	4.76000
ρ_4	3.53000	3.53000	3.53000	3.53000	3.53000	3.53000
RMS=	0.13211	0.03995	0.04322	0.04031	0.04040	0.04012

tion of layers very similar to that of case Xp2. In Xp3, however, ρ_1 is replaced in the group of active parameters. In Xp3 no limiting configuration is reached in the course of five revisions, although all corrections except the second have a standard deviation that is virtually identical to that reached as a limiting value in Xp1 and Xp2.

Discussion

The theoretical cases Th1, Th2, and Th3 are a general test of this inversion method. The results demonstrate the feasibility of curve fitting by means of the period equation in cases of limited complexity for which good data are available. In all three experiments there is a strong tendency to converge to the known, correct answer, although the results in Th3 are erratic except very near the correct solution. In Th3 all corrections except the first represent an improvement in s; in Th1 and Th2 s decreases consistently to a limiting value of the order of 10^{-5} km/sec. Convergence in Th3 is not as rapid as in Th1 and Th2, indicating that trouble might be expected in a similar experimental problem for a two-layered crust.

The results in Th1, Th2, and Th3 strongly suggest a unique relationship between the respective dispersion curves and the velocity and density structures involved. The apparent disagreement of this result with Knopoff's conclusion, quoted above, may probably be explained by the fact that, in these inversion experiments, variations of density and shear velocity are permitted only in the form of one or two discontinuities rather than as arbitrary functions of depth, as in Knopoff's treatment of the problem. This distinction between the two methods may be of practical significance, however. If one is satisfied to determine shear velocity in terms of constant values applied to homogeneous regions between a few discontinuities, the results of the present experiments show in principle that density structure can be obtained simultaneously in the same form. This approach is worth while, since the ability to characterize the crust or upper mantle of a region objectively in terms of the shear velocities and densities of only a few layers would be a definite advance in the present stage of seismic exploration.

Finally, the theoretical problems show specifically how the inversion program responds in problems similar to the New York–Pennsylvania problem. However, they differ from the usual experimental situation in that the data correspond exactly to a particular set of homogeneous layers and are not affected by experimental scatter. Nevertheless, these problems indicate that a simple approximation of crustal structure may be derived by applying the program to the experimental New York–Pennsylvania dispersion data.

In calculations on the data of paper I, the simplest configuration that can account for dispersed Rayleigh waves, i.e., a one-layered crustal model with homogeneous mantle, was used in Xp1. The standard deviation obtained in the fifth correction is the lowest in any experiment with the New York–Pennsylvania data, and it therefore confirms the statistical validity of the result. The crustal density obtained does not fall in the range of physical values, however, and the velocity structure is inconsistent with Katz's results. On the other hand, a velocity structure in much better agreement with that derived by Katz from shear waves is obtained in Xp2 by the addition of a channel of the Lehmann type [see *Dorman et al.*, 1960] represented by fixed layering in the mantle. This suggests that the model of the mantle low-velocity channel in Xp2 is approximately correct, although these data by themselves are not sufficient to eliminate the possibility of a different channel configuration, i.e., a channel of the Gutenberg type. Neglect of the channel effect on structural interpretations of dispersion data was mentioned in paper I as a source of serious errors in crustal studies in which the dispersion method is used.

A further step, taken in Xp3, is an attempt to measure simultaneously one additional parameter, crustal density, ρ_1. In Xp2, $\rho_1 = 2.70$ g/cm^3 was chosen arbitrarily. However, in problem Xp3, where ρ_1 is allowed to vary, it appears that the number of active parameters is too large for a unique solution to be obtained in this configuration with the available data. Other attempted inversion interpretations involving additional layers and/or additional active parameters behaved in a similar way. Table 6 shows the wide range of statistically valid solutions that were obtained in the various corrections of Xp3. The method of plotting these data in Figure 1 (solid circles) shows that these solutions form a linear distribution. The solution of Xp2, shown by the open circles in Figure 1, is unique with respect to its own constraints and is a member of the more general class of solutions of Xp3. The one-dimensional nature of the distribution of solutions in Figure 1 is emphasized by the fact that the points which fall farthest from the linear trends, those of the second correction of Xp2, also have an s which departs considerably from the very closely grouped values associated with the other points. This behavior suggests that three is the maximum number of parameters that can be evaluated by inversion in this configuration without data of better quality.

Failure of convergence in Xp3 is in contrast with the results of Th2, where convergence of a

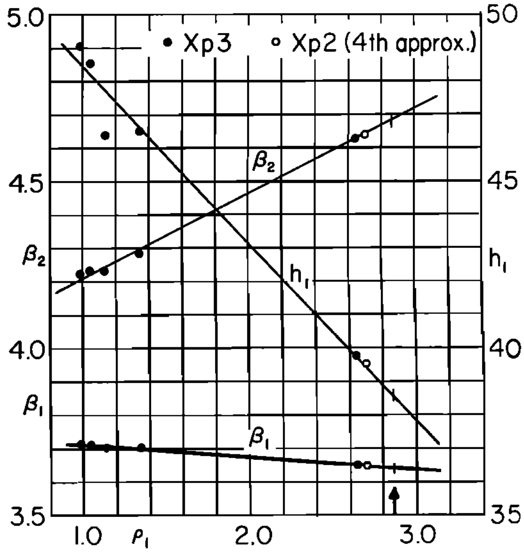

Fig. 1. Parameters h_1, β_1, β_2 of case Xp3, corrections 1 through 5, and case Xp2, correction 4, plotted against the parameter ρ_1. β_1 and β_2 are given in km/sec; h_1 is km; and ρ_1 is in g/cm^3. Each of these cases is a good solution. The arrow indicates parameters of case Xp3A, also a good solution, in which β_2 agrees with Katz's refraction observations.

four-variable problem is obtained with a virtually identical data spectrum and configuration of layers. The difference in behavior between the two problems is probably due to scatter of the data in Xp3 which removes the sharpness of the unique minimum s value found by the inversion program in Th2. We also note that *Brune and Dorman* [1963], using data of better quality, have obtained inversion results on the Canadian shield which delineate greater detail and complexity of layering than are possible with the New York–Pennsylvania data. These facts suggest that results obtained from the New York–Pennsylvania data do not represent absolute performance limitations of this inversion method. On the contrary, our results are limited mainly by the input data.

Presumably, any set of four parameters selected by drawing a vertical line that intersects the β_2, β_1, and h_1 curves and the ρ_1 axis in Figure 1 would define an equally valid solution of the dispersion data from a statistical point of view. Therefore, by associating Katz's observation of the apparent velocity of *Sn*, 4.68 to 4.69 km/sec with the parameter β_2 in Figure 1, we ob-

TABLE 7. Comparison of Parameters of Crustal Models for the New York–Pennsylvania Area

	Katz, Milroy Profile (shear waves)	Katz, Tahawus-west Profile (shear waves)	Inversion of Rayleigh Wave Dispersion Data (case Xp3A)
h_1	36.3 $\pm$ 4.3 km	38.4 $\pm$ 3.6 km	38.6 km
β_1	3.61 $\pm$ 0.01 km/sec	3.62 $\pm$ 0.03 km/sec	3.64 km/sec
ρ_1	...	...	$0.866\rho_2$
β_2	4.69 $\pm$ 0.03 km/sec	4.68 $\pm$ 0.02 km/sec	4.685 km/sec (chosen)
Mantle heterogeneity	No information	No information	Low shear velocity channel required

tain a model which agrees well with surface wave and body wave data. The corresponding abscissa, $\rho_1 = 2.86$ g/cm, is identified by the arrow labeled case Xp3A in Figure 1. Values of β_1 and h_1, 3.64 km/sec and 38.6 km, respectively, which correspond to Xp3A in Figure 1 are in very good agreement with Katz's values, as shown in Table 7. Therefore, the density ratio $\rho_1/\rho_2 =$ 0.866, obtained without reference to gravity observations, represents the best agreement between refraction results and the surface wave data. This density ratio, though not precisely measured, is in good agreement with the result $\rho_1 = 2.84$, $\rho_2 = 3.27$, or $\rho_1/\rho_2 = 0.869$ obtained by *Worzel and Shurbet* [1955] from analysis of gravity observations.

Figure 2 shows that Xp3A does indeed fit the data of paper I in a satisfactory way, as inferred from Figure 1. For Xp3A, $s = 0.040$ km/sec, which is satisfactory in comparison with other solutions obtained in Xp1, Xp2, and Xp3. From Figure 2, Xp3A appears to be in much better agreement with the data than case 8123 (the final solution offered in paper I) for which we now calculate $s = 0.057$ km/sec. Table 8 gives the complete list of parameters of case Xp3A used in calculating the corresponding theoretical curves in Figure 2. In Xp3A, α_1 and α_2 were chosen to be consistent with Katz's compressional velocity data. The parameters of the mantle layers are those used by *Dorman et al.* [1960]. Values of mantle shear wave velocity depend on data of *Lehmann* [1955, 1961], and the mantle density structure is interpolated from Bullen's model A [*Bullen*, 1947].

The crustal model Xp3A has the great advantage of simplicity. From experience with Xp3 it is clear that the number of parameters needed to characterize a three-layered crust, the model assumed in paper I, cannot be evaluated uniquely from the present dispersion data even with information from Katz's refraction experiments. Thus the present inversion method has permitted us to estimate realistically the amount of structural information which can be deduced uniquely from present body wave and surface

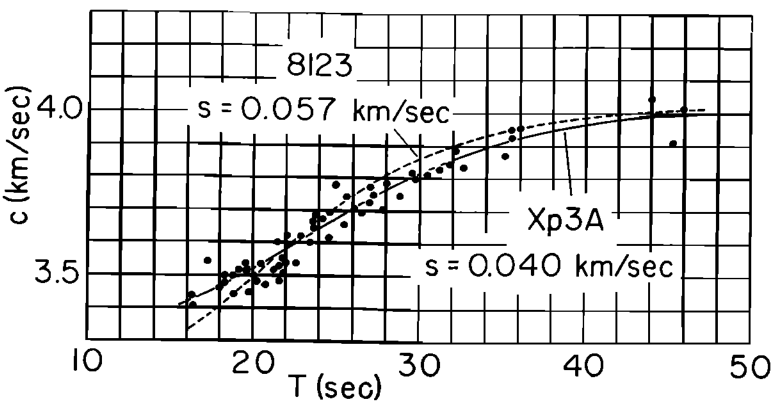

Fig. 2. Dispersion curves for case Xp3A and case 8123 plotted against dispersion data of *Oliver et al.* [1960] (solid circles).

TABLE 8. Parameters of Cases 8123 and Xp3A

h, km	α, km/sec	β, km/sec	ρ, g/cm^3
Case 8123, s = 0.057 km/sec			
1.89	3.98	2.30	2.34
25.11	6.15	3.55	2.817
9.00	6.58	3.80	2.922
84.00	8.14	4.70	3.30
100.00	8.17	4.30	3.44
Infinite	8.49	4.76	3.53
Case Xp3A, s = 0.040 km/sec			
38.6	6.15	3.64	2.86
84.0	8.14	4.685	3.30
100.0	8.17	4.30	3.44
Infinite	8.49	4.76	3.53

wave data and to provide satisfactory numerical values within this framework. The values of Table 8, therefore, may be regarded as the most detailed model of the crust and upper mantle that is justified by present data. The fact that Katz failed to identify consistent P_2 or S_2 and thereby the presence of a deeper high-speed crustal layer favors the one-layer crustal hypothesis. When further data become available, it may be necessary to revise this model. In particular, short-period normal-mode data (T < 10 sec), not now available, should reflect the presence of a surface low-velocity region in the upper crust.

Mutually consistent results with respect to crustal thickness and shear velocity distribution from the analysis of surface waves and refracted shear waves are not surprising, since the properties of surface waves are determined mainly by the shear properties of the rocks. However, this calls further attention to the systematic difference in crustal thickness obtained by the compressional and shear wave refraction methods. As noted by Katz and others, the shear wave travel-time curves often give a crustal thickness about 10 per cent greater than the compressional-wave travel-time curves. Katz's hypothesis that the gradient of compressional velocity with respect to depth is algebraically greater than the gradient of shear velocity with respect to depth is a reasonable explanation of the facts. The present analysis of surface waves yields no evidence of the existence of small velocity gradients in the crust.

The inverse of the slopes of the curves in Figure 1 are a measure of the relative precision with which the various parameters are determined when a particular solution is selected from the linear family of surface wave solutions. Further insight into the relationships expressed in Figure 1 is obtained from the partial derivative relationships shown in Figure 3. The partial derivatives from which these curves were drawn were computed as indicated by (3). Thus each curve gives, as a function of period, the phase velocity increment in kilometers per second which would be produced by a 10 per cent increment in the corresponding parameter. The distribution of the experimental data points along the period axis is indicated by the arrows at the bottom of the figure. β_1 is clearly the most significant and well determined of the four variables, as is shown by the height of curve 2 and the concentration of data points in the region of its maximum, as well as by the small slope of the β_1 curve in Figure 1. Curve 1, Figure 3, shows that the distribution of experimental data points is favorable for the determination of crustal thickness, although the height of curve 1 is somewhat low, with respect to curves 2 and 4, for precise determination. Curve 3 is relatively low throughout, and it decreases rapidly toward the short periods where

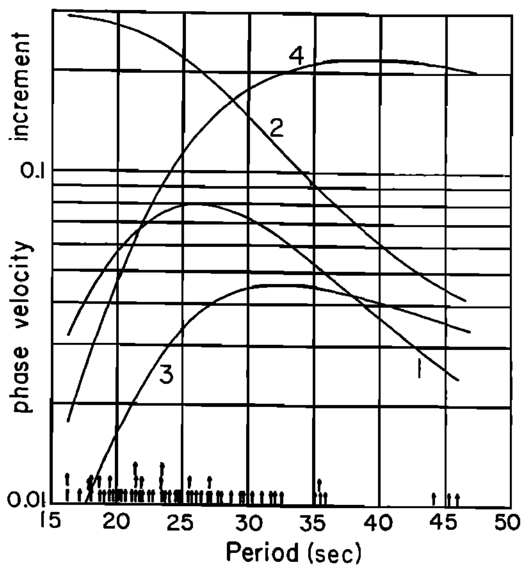

Fig. 3. Phase velocity increment in kilometers per second versus period in seconds for hypothetical changes of 10 per cent in the active parameters of case Xp3, first correction. Curve 1, $-0.1\ h_1\ \partial c/\partial h_1$; curve 2, $0.1\beta_1\ \partial c/\partial \beta_1$; curve 3, $-\ 0.1\rho_1\ \partial c/\partial \rho_1$; curve 4, $0.1\beta_2\ \partial c/\partial \beta_2$.

most of the experimental data are concentrated. Therefore, ρ is determined with relatively poor precision. Figure 3 suggests that additional data for periods between 30 and 50 seconds would improve the estimate of ρ_1. Clearly a unique interpretation in terms of more layers in the same depth range can only be obtained if the experimental data contain more information in the form of reduced scatter, broader frequency coverage, and additional higher modes.

Because of the similarity in the configuration of the layers in problems Th2 and Xp3, the curves of Figure 3, based on Xp3, apply very nearly to Th2 as well. Therefore, by comparing the residuals and the corresponding configuration of layers in Table 2 with the curves of Figure 3, one can examine the operation of the inversion program and gain some insight into the effects of the various parameters.

Conclusions

Fitting curves by least mean squares with an electronic computer is a useful technique for obtaining the best-fitting set of values for elastic parameters of a layered earth model with respect to particular phase velocity dispersion data for seismic surface waves. This method has been applied to the problem of crust-mantle structure in the New York–Pennsylvania area by reinterpreting the Rayleigh wave dispersion data of Oliver, Kovach, and Dorman. Related seismic refraction data and near-earthquake data were used concurrently. The calculations show that the surface wave dispersion data are mutually consistent with velocity structures for the area derived by Katz from seismic refraction and by Lehmann from travel-time studies of nearby earthquakes. In addition, the density calculated by seismic inversion is consistent with Worzel and Shurbet's results that were based on gravity data and the assumption of isostatic compensation. A summary of results to date on shear velocity and density structure in the area is given in Table 7. Table 8 gives a complete list of the parameters of case Xp3A, the model offered as the best current solution of pertinent surface wave and body wave data. Figure 2 shows the good agreement between the dispersion data of Oliver, Kovach, and Dorman and the dispersion curve for case Xp3A.

Questions concerning the uniqueness of a particular solution or the amount of information about structure obtainable from certain data can be answered quickly by experimentation with various configurations of layers by taking advantage of high-speed computers. Calculated partial derivatives, such as those shown in Figure 3, are valuable for promoting the understanding of the functional relations between dispersion data and a corresponding layered structure. The variable parameters can then be evaluated rapidly by calculation. The convenience and power of the least-squares inversion method has enabled us to find a solution fitting all available data on the New York–Pennsylvania area. It is simpler and it fits the data better than previous solutions.

An increase in the number of modes in which experimental data are available, a broadening of the spectrum in each mode, and a decrease in the scatter of the data are factors which improve the performance of the inversion calculation. When data of better quality than the New York–Pennsylvania data are available, a more detailed structural interpretation, covering a greater range of depths, can be obtained by numerical inversion.

Acknowledgments. This work was supported by the Air Force Cambridge Research Laboratories under contract AF 19 (604) 8375 and 7376 with Lamont Observatory. The Institute for Space Studies and the Theoretical Division, Goddard Space Flight Center, aided with the necessary computations.

We are indebted to Dr. Jack Oliver for useful discussions and criticism.

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(Manuscript received May 12, 1962;
revised September 17, 1962.)